

STATUS OF HIGH ENERGY FORWARD ELASTIC SCATTERING

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Abstract

We present the results of fitting all data for pp and $\bar{p}p$ scattering at $\sqrt{s} \geq 9.7$ GeV and up to the collider energy with various analytic parametrizations of the elastic forward scattering amplitudes based on the derivative dispersion relation. It is found that the model containing the Pomeron and Reggeon terms with 8 parameters has the most preferred $\chi^2/d.o.f = 1.3$, while the Donnachie and Landshoff model with 5 parameters has $\chi^2/d.o.f = 2.16$ for a data set of 111 experimental points. The current data however make no clear preference between the ℓns and $\ell n^2 s$ type Pomerons.

I. Complete Survey of Experiments

It is crucial to select as complete a set of data as possible. No experimental group is to be left out, nor any errors to be reduced to account for paucity of higher energy results. This type of analysis has been done on different sets of data depending only on the lowest value of \sqrt{s} allowed: 9.7 GeV vs. 5 GeV¹⁾. This report deals exclusively with the data set containing 111 experimental points for the lowest value of $\sqrt{s} = 9.7$ GeV distributed as follows: 58 values of σ_T , 22 for $\bar{p}p$ and 36 for pp , and 53 values of ρ , 12 for $\bar{p}p$ and 41 for pp . We included the new datas from UA4²⁾, CDF³⁾, and E710⁴⁾. Because the majority of precise data is at lower energies ($\sqrt{s} < 63$ GeV), we should expect a detailed Regge parametrization. Also because the newest higher energy experimental data are closer to standard theoretical expectations, we should expect several theoretical models to do well.

II. Models for the Elastic Forward Scattering Amplitude

Though we have in principle the exact theory of the strong interactions, QCD, which can describe, based on the perturbative calculation, the hadron interactions at short distances, the interactions at large distance, i.e., near forward scattering can not reliably be calculated with the perturbative QCD. On the other hand, phenomenological models for high energy scattering based on general principles such as unitarity, analyticity and crossing-symmetry, have proven to be successful in understanding or predicting the high-energy behavior of the hadronic scattering amplitude⁵⁾. Two such examples are analytic amplitude models as solutions of the Derivative Dispersion Relation (DDR)⁶⁾ derived from the Sommerfeld-Watson-Regge representation (SWR) and comprehensive eikonal models⁷⁾, based on QFT expectations or the parton model.

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In this report we will concentrate on the analytic amplitude models derived from DDR. The crossing-even and odd amplitudes F_{\pm} for pp , $\bar{p}p$ scattering are defined by

$$F_{\pm} = \frac{1}{2}(F_{pp} \pm F_{\bar{p}p}) \quad (1)$$

where the normalization condition is given by

$$\sigma_T = \frac{1}{s} \text{Im} F(s, t=0) \quad (2)$$

to satisfy the optical theorem.

A Regge pole at $J = \alpha_R(t)$ in the complex j -plane gives

$$F_+^k(s, t) = C_+^k(t) [i - \cot(\frac{\pi}{2} \alpha_+^k(t))] s^{\alpha_+^k(t)} \quad \text{if } C \text{ is even} \quad (3)$$

$$F_-^k(s, t) = -C_-^k(t) [i + \tan(\frac{\pi}{2} \alpha_-^k(t))] s^{\alpha_-^k(t)} \quad \text{if } C \text{ is odd} \quad (4)$$

If the Regge pole is Exchange-Degenerate, we have

$$C_+^k = C_-^k, \quad \alpha_+^k = \alpha_-^k \quad (5)$$

Clearly Regge pole model such as the Donnachie-Landshoff model satisfies DDRs. A small $\alpha(0) - 1 = 0.08$ is consistent with the slow increase of $\sigma_T(s)$ ⁸⁾, but would eventually be in conflict with the unitary condition, i.e., the Froissart bound $\sigma_T(s) \leq C(\ln s)^2$. A fully unitary theory must satisfy multiparticle unitarity in both s - and t -channels. Also the Pomeronchuk theorem, i.e., $\sigma_T^{\bar{p}p}(s)/\sigma_T^{pp}(s) \rightarrow 1$ as $s \rightarrow \infty$, can be proven rigorously only if total cross-sections increase with energy⁹⁾.

If the total cross-section increases with energy, the leading j -plane singularity of F_+ is at $j = 1$ in the forward direction. One can then write an analytic parametrization for the contribution of this asymptotic part from the derivative dispersion relation

$$F_+^{p2}(s, o) = is[A_+ + B_+(\ln \frac{s}{s_+} - i\frac{\pi}{2})^2] \quad (6)$$

if $\sigma_T(s)$ behaves like $(\ln s)^2$ at high energies and

$$F_+^{p1}(s, o) = is[A_+ + B_+(\ln \frac{s}{s_+} - i\frac{\pi}{2})] \quad (7)$$

if σ_T increase with energy as $\ln s$. On the other hand, the odd-signatured counter part of the Pomeron can also be constructed by that the difference $\Delta\sigma = \sigma_T^{\bar{p}p} - \sigma_T^{pp}$ does not necessarily vanish asymptotically. This implies the leading j -plane singularity of F_- is also at $j = 1$ in the forward direction and one can get the maximal Odderon amplitude from the DDR

$$F_-^o(s, o) = s[A_- + B_-(\ln \frac{s}{s_-} - i\frac{\pi}{2})^2] \quad \text{if } \Delta\sigma \sim \ln s \quad (8)$$

$$F_-^o(s, o) = s[A_- + B_-(\ln \frac{s}{s_-} - i\frac{\pi}{2})] \quad \text{if } \Delta\sigma \rightarrow \text{const.} \quad (9)$$

Equation (8) is the maximal Odderon amplitude if F_+ satisfies Eq.(6) because $\Delta\sigma \leq (\ln s)^{\beta/2}$ asymptotically if the cross-section increase as $(\ln s)^{\beta}$ with $\beta \leq 2$.

The asymptotic analytic amplitude model can then be constructed in various form of

$$F_+ = F_+^{p_i}(s, o) + \sum_k F_+^{(k)}(s, o) \quad (10)$$

$$F_- = F_-^o(s, o) + \sum_k F_-^{(k)}(s, o) \quad (11)$$

where $F_{\pm}^{(k)}$ represents the Regge amplitudes.

III. Comparison with Experiment

We now present the results of our comprehensive χ^2 -fits to 111 high energy data points by theoretical models with and without the Odderon terms along with the predictions at the LHC energy.

Depeding on the choices of various terms from the expressions (10) and (11), we studied the following 10 models:

(a) Model A1: The Block-Kang-White model¹⁾, $P_1 + RND_+ + RND_-$, where P_1 is the ℓns -type Pomeron term and RND_{\pm} represent non-degenerate Regge terms. Since $A_+ - B_+ \ell ns_+$ is fixed, this model has 6 (=3+2+2-1) free parameters.

(b) Model A2: Modification of Model A1 by replacing P_1 by the $(\ell ns)^2$ -type Pomeron term, $P_2 + RND_+ + RND_-$ with 7 parameters.

(c) Model A3: $P_2 + RD$ with 5 parameters, where RD is the exchange degenerate Regge term.

(d) Model B1: $P_1 + RD + RND_+ + RND_-$ with 8 free parameters.

(e) Model B2: Modification of Model B1 by replacing P_1 by P_2 , i.e., $P_2 + RD + RND_+ + RND_-$ with 9 free parameters.

(f) Model E1: The maximal Odderon model, $P_2 + O + RD + RND_+ + RND_-$ with 12 free parameters.

(g) Model E2: The maximal Odderon model with one exchange degenerate Regge term, $P_2 + O + RD$ with 8 parameters.

(h) Model E3: The maximal Odderon model without the exchange degenerate Regge term, $P_2 + O + RND_+ + RND_-$ with 10 parameters.

(i) Model F1: The model with the bare Pomeron plus the nondegenerate Regge terms,

$$\begin{aligned} C_1 s^{A_1} + C_2 s^{-A_2} + C_3 s^{-A_3} & \quad \text{for } \sigma_{p\bar{p}}, \\ C_1 s^{A_1} + C_2 s^{-A_2} - C_3 s^{-A_3} & \quad \text{for } \sigma_{pp}. \end{aligned}$$

This model has 6 parameters.

(j) Model F2: The Donnachie-Landshoff (DL) Model⁸⁾, which is a modification of Model F1 by setting $A_2 = A_3$. This model is similar to Model A3 and has 5 free parameters.

The results of the fits for the parameters are given below in Table along with the predictions of ρ and σ_T at LHC ($\sqrt{s}=14$ TeV). Fig. 1 shows fits to Model B1 and Fig. 2 exhibits the fits to Model F2, the Donnachie-Landshoff Regge model.

IV. Conclusions

(a) Model B1, $P_1 + RD + RND$, and Model B2, $P_2 + RD + RND$, have the most preferred fits with $\chi^2/d.o.f = 1.3$ to the data for $\sqrt{s} \geq 9.7$ GeV. There are, however, no significant differences between the two models: the current data are consistent with either ℓns or $\ell n^2 s$ increase of σ_T .

(b) All of the models considered above based on the asymptotic analytic amplitudes reproduce the data more or less equally well, i.e., $\chi^2/d.o.f \simeq 1.3 \sim 1.45$, considering the quality of the data from different experiments. In particular, the maximal Odderon models E1 and E2 give comparable fits as those of Models B1 and B2 so that there seems to be little support for the

Odderon in the current data.

(c) The Regge model F2 has relatively high $\chi^2/d.o.f$ but the Model F1, which is a modified version of F2 by relaxing the exchange degeneracy assumption, has significantly improved and comparable $\chi^2/d.o.f$ value as those of Model A1 or A2.

(d) For the asymptotic behavior of $\sigma_T(s)$ extracted from this study and a comparison with the phenomenological fits made in the 1994 Review of Particle Properties¹⁰⁾, see Kang's talk¹¹⁾.

(e) We denote that the coefficient C 's of exchange degenerate Regge terms have values with huge errors in Model B1 and B2: 6042.5 ± 2261.6 and 8857.2 ± 7424.6 , respectively. Thus we should need more low energy data by lowering the cut-off below 9.7 GeV, if their errors are to be reduced and improved.

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V. References

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Figure Captions

Fig. 1 Model (B1) fits to (a) σ_T and (b) ρ

Fig. 2 Model (F2) fits to (a) σ_T and (b) ρ

Modles	A1	A2	A3	F1	F2
A_+	21.241	28.286	39.062		
B_+	6.8241	0.22793	0.36909		
$\ln \sqrt{s_+}$	3.9465	0.19882	2.2001		
A_-	0.	0.	0.		
B_-	0.	0.	0.		
$\ln \sqrt{s_-}$	0.	0.	0.		
C_+	104.93	51.914	0.	$C_1 : 18.952$	22.782
α_+	0.80213	0.62470		$A_1 : 0.093272$	0.076403
C_-	34.498	35.544	0	$C_2 : 62.182$	$C_2 + C_3 : 98.278$
α_-	0.45051	0.44443		$A_2 : 0.35989$	0.47349
C	0.	0.	37.387	$C_3 : 35.677$	$C_2 - C_3 : 52.055$
α			0.43500	$A_3 : 0.55633$	
χ^2	146.0307	142.4876	154.1655	142.8956	229.2668
$\chi^2/d.o.f$	1.39077	1.37007	1.45439	1.36091	2.16289
$\rho_{p\bar{p}} (546)$	0.1283	0.1361	0.1507	0.1395	0.1182
. (1800)	0.1197	0.1351	0.1543	0.1448	0.1199
. (14000)	0.0994	0.1244	0.1446	0.1471	0.1205
$\rho_{pp} (546)$	0.1275	0.1354	0.1498	0.1388	0.1163
. (1800)	0.1196	0.1350	0.1542	0.1446	0.1194
. (14000)	0.0994	0.1244	0.1446	0.1471	0.1205
$\sigma_{p\bar{p}} (546)$	62.10	62.18	63.06	62.11	59.94
. (1800)	75.09	76.46	79.57	77.01	71.70
. (14000)	100.1	107.4	117.8	112.5	98.00
$\sigma_{pp} (546)$	62.03	62.12	63.00	62.05	59.82
. (1800)	75.07	76.44	79.55	76.99	71.66
. (14000)	100.1	107.4	117.8	112.5	97.99
Modles	E1	E2	E3	B1	B2
A_+	28.830	39.269	29.330	6.4816	28.738
B_+	0.23280	0.36182	0.23010	8.2255	0.24247
$\ln \sqrt{s_+}$	0.26108	2.1936	0.29304	4.1055	0.43383
A_-	- 0.47475	12.002	0.28456	0.	0.
B_-	- 0.12010	- 0.00040921	- 0.036942	0.	0.
$\ln \sqrt{s_-}$	6.1277	89.530	6.0487	0.	0.
C_+	157.28	0.	54.878	126.05	42.091
α_+	0.58499		0.59521	0.85242	0.66971
C_-	164.03	0.	112.94	25.689	25.249
α_-	0.47052		0.11584	0.49582	0.49882
C	- 103.03	44.817	0.	6042.5	8857.2
α	0.57285	0.37601		-1.1661	-1.2384
χ^2	133.2673	148.3081	134.1526	133.9024	134.1805
$\chi^2/d.o.f$	1.34613	1.43988	1.32824	1.30002	1.31550
$\rho_{p\bar{p}} (546)$	0.1343	0.1374	0.1297	0.1337	0.1384
. (1800)	0.1499	0.1395	0.1344	0.1272	0.1380
. (14000)	0.1824	0.1300	0.1385	0.1076	0.1273
$\rho_{pp} (546)$	0.1394	0.1592	0.1412	0.1325	0.1371
. (1800)	0.1222	0.1653	0.1349	0.1269	0.1377
. (14000)	0.07015	0.1567	0.1100	0.1076	0.1272
$\sigma_{p\bar{p}} (546)$	62.21	63.06	62.28	62.32	62.25
. (1800)	75.99	79.28	76.30	76.09	76.82
. (14000)	106.0	116.8	106.8	103.6	108.8
$\sigma_{pp} (546)$	63.01	62.60	62.39	62.23	62.16
. (1800)	78.28	78.85	76.97	76.06	76.79
. (14000)	111.2	116.4	108.4	103.6	108.8

Table

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